### MA110 - Engineering Mathematics-1
Problem Sheet - 9

Double Integrals in Polar Coordinates

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1. Use double integrals in polar coordinates to compute the volume V of a solid bounded by the spherical surface $x^2 + y^2 + z^2 = 4a^2$ and the cylinder $x^2 + y^2 - 2ay = 0$.

2. Set up double integral in polar coordinates in the order $d\theta\, dr$ to find the area of the region between the circles $r = 2\sin\theta$ and $r = 4\sin\theta$.

3. Evaluate $\iint \dfrac{r}{\sqrt{a^2 + r^2}}\, dr\, d\theta$ over one loop of the lemniscate $r^2 = a^2 \cos 2\theta$.

4. Use double integrals in polar coordinates to find the area of the region enclosed by the positive x-axis and spiral $r = \frac{4}{3}\theta,\, 0 \le \theta \le 2\pi$. The region looks like a snail shell.
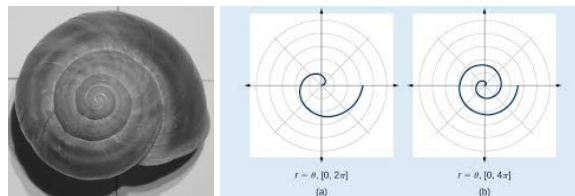
5. Use double integrals in polar coordinates to find the average distance from a point $P(x, y)$ in the disk $x^2 + y^2 \le a^2$ to the origin.

6. Use double integrals in polar coordinates to find the area of the region which lies inside the circle $x^2 + (y - 1)^2 = 1$ but outside the circle $x^2 + y^2 = 1$.

7. A flat plate is in the shape of the region R defined by the inequalities $x^2 + y^2 \le 4,\, 0 \le y \le 1$ and $x \le 0$. The density of the plate at the point (x, y) is $-xy$. Use double integrals in polar coordinates to find the mass of the plate.

8. Evaluate following integrals using polar co-ordinates:

(a) $\displaystyle\int\limits_{1}^{2} \int\limits_{0}^{\sqrt{2x - x^2}} (x^2 + y^2)^{-1/2} dy\, dx$ (b) $\displaystyle\int\limits_{0}^{1} \int\limits_{0}^{\sqrt{1 - y^2}} \sin(x^2 + y^2)\, dx\, dy$

9. Find the volume of the wedge cut from a right circular cylinder of radius a by a plane through a diameter of its base and making an angle $\alpha(0 < \alpha < \pi/2)$ with the base.

10. Evaluate $\iint\limits_{S} |\sin(\sqrt{x^2 + y^2})|\, dA$ where S is the region inside $x^2 + y^2 = 4\pi^2$.

11. Using polar coordinates, find the volume of the solid bounded above by $2x^2 + 2y^2 + z^2 = 18$ below by $z = 0$ and laterally by $x^2 + y^2 = 4$.

12. Integrate $f(x,y) = \dfrac{1}{(1+x^2+y^2)^2}$ over first quadrant of $xy-$plane.

13. Find the area enclosed by one leaf of the rose $r = 12\cos 3\theta$.

14. Consider the circle $r = 1$ and the cardioid $r = 1 + \sin\theta$. Find the area: (a) Inside the cardioid and outside the circle, (b) Inside the cardioid and the circle, (c) Outside the cardioid and inside the circle.

15. Using polar form, evaluate $\displaystyle\int_0^\infty e^{-x^2}dx$

16. Find the average value of: (a) $f(x,y) = x\cos xy$ over the region $R : 0 \le x \le \pi,\ 0 \le y \le 1$, (b) $f(x,y) = xy(x+y)$ over R, where R is bounded by $x^2 = y$ and $y = x$.

17. The triangle is an isosceles triangle (as in figure) with its base passes through the centre of the circumscribing circle with diameter 40 cm. Calculate the area of the shaded region.

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